Use of Euler potentials for describing magnetosphere-ionosphere coupling


Received 30 November 2005; revised 16 March 2006; accepted 19 April 2006; published 26 July 2006.

[1] We present a general formulation of the basic equations of large-scale magnetosphere-ionosphere coupling in terms of Euler potentials and describe a specific numerical implementation of this formulation in the context of the Rice Convection Model (RCM). When written in terms of Euler potentials, both the Vasyliunas magnetosphere-ionosphere coupling equation and the expression for bounce-averaged adiabatic drift assume particularly elegant forms, while the equation for the conservation of ionospheric current is only slightly more complicated to solve than the corresponding formula for a dipole case. For simplicity, large-scale models of convection in the inner magnetosphere have typically assumed strict symmetry between the northern and southern hemispheres, explicitly assuming the internal planetary magnetic field to be a dipole aligned with Earth’s rotation axis and oriented perpendicular to the solar wind flow velocity. These approximations have precluded the realistic treatment of ionospheric longitude and seasonal effects as well as dipole-tilt and IMF-$B_y$-penetration effects in the magnetosphere. We present a scheme for constructing an Euler-potential-based computational mesh, in which the Euler potential $\alpha$ is set to zero at the dip equator for a reference altitude of 90 km, and $\beta$ lines in the northern ionosphere follow lines of constant centered dipole magnetic longitude but are spaced equally in terms of total latitude-integrated magnetic flux. Properties of the Euler-potential-based grid are illustrated using an IGRF model for the Earth’s internal field. Our procedure yields an Euler-potential-based grid that covers the entire ionosphere, except for the southern polar cap and cusp.


I. Introduction

[2] Vasyliunas [1970] first presented a computational scheme for self consistently calculating the electrodynamic and plasma processes coupling the inner magnetosphere and ionosphere. Expressed in general form, this scheme consists of three sets of relations:

[3] 1. The first relation is an expression relating plasma pressure gradients in Earth’s magnetic field to field-aligned currents linking the magnetosphere and ionosphere. Neglecting inertial drift currents compared to gradient/curvature drift currents and assuming an isotropic plasma pitch angle distribution within a magnetic flux tube, this relation (termed the Vasyliunas equation) can be written as

$$\frac{J_{\parallel n}}{B_n} - \frac{J_{\parallel s}}{B_s} = \mathbf{b} \cdot \nabla V \times \nabla P $$

(1)

where $J_{\parallel n}$ and $J_{\parallel s}$ are the density of Birkeland current along the magnetic field direction just above the northern and southern ionospheres, respectively, $\mathbf{b}$ is a unit vector along the magnetic field $V = \int \frac{ds}{B}$ is the volume of a tube of unit magnetic flux, and $P$ is magneto-spheric particle pressure [e.g., Wolf, 1983; Heinemann and Pontius, 1990]. The right side of (1) can be calculated anywhere on the field line.

[4] 2. The second set of relations consists of expressions for ionospheric current continuity. If one makes the conventional assumption that the induction electric field in the ionosphere is negligible, so that the electric field there is simply $\mathbf{E} = -\nabla \Phi$, then the standard expression for ionospheric conduction current is

$$\mathbf{J} = \sigma_p (-\nabla \Phi + \mathbf{v} \times \mathbf{B}) - \sigma_{\parallel} (-\nabla \Phi + \mathbf{v} \times \mathbf{B}) \times \mathbf{b}$$

(2)

where $\mathbf{b}$ is a unit vector in the direction of the magnetic field, and $\mathbf{v}$ is the neutral wind velocity. Setting the divergence in $\mathbf{J}$ equal to the computed field-aligned current yields an elliptic partial differential equation that can be solved for the electric potential distribution in the ionosphere, given the electrical conductance and neutral wind fields. Integrating (2) over the conducting region of each field line and requiring that the divergence of horizontal ionospheric conduction current be balanced by

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0148-0227/06/2005JA011558$09.00
the field-aligned current from the magnetosphere leads to the relation

$$\nabla_h \cdot \left[ \Sigma \cdot (-\nabla \Phi) \right] + w = J_i \sin I$$  \hspace{1cm} (3)

where $\Sigma$ is the conductance tensor representing single-hemisphere field-line-integrated conductivities, $w$ represents the field line integrals of products of conductivity and $v \times B$ (see, e.g., Forbes and Harel [1989] for explicit approximate forms) and $I$ is the magnetic dip angle. Equation (1) is usually substituted in (3) using the assumption that $J_i/B$ is antisymmetric between the two hemispheres.

The third set of relations consists of expressions describing the bounce-averaged, energy-dependent adiabatic drift of magnetospheric plasma. In convection and ring current models, the bounce-averaged drift equation is usually written in a form like

$$v_D = E_{ind} \times B \over B^2 + B \times \nabla H \over qB^2$$  \hspace{1cm} (4)

Here $E_{ind}$ is the induction electric field, the total electric field is $E = E_{ind} - \nabla \Phi$, and $q$ is the charge. The Hamiltonian $H$ is given by

$$H = q\Phi + W_K$$  \hspace{1cm} (5)

where $W_K$ is the kinetic energy of a particle written as a function of position, time, and the appropriate adiabatic invariants. Specifically, when the particles are assumed to be undergoing strong, elastic pitch angle scattering, as in the RCM, we set

$$W_K = \lambda V^{-2/3}$$  \hspace{1cm} (6)

where $\lambda$ is the isotropic energy invariant, and $V = \int ds/B$ is the volume occupied by a flux tube carrying one unit of magnetic flux, between the southern ionosphere and the northern. For the less restrictive case where the model particle distribution is organized in terms of the first two adiabatic invariants, as in most ring current-particle models, including specifically the Comprehensive Ring Current Model (CRCM) [Fok et al., 2001], $W_K$ is taken to be a function of the first and second adiabatic invariants $\mu$ and $J_l$, as well as position and time.

The plasma distribution is advanced in time using

$$\frac{\partial f}{\partial t} + v_D \cdot \nabla f = S - L$$  \hspace{1cm} (7)

where $S$ and $L$ represent source and loss terms for the magnetospheric plasma flux tube (e.g., upflowing particles, precipitation, and charge exchange), and $f$ is a function of appropriate adiabatic invariants, position, and time. We note that the derivation of (1) requires neglect of inertial currents compared to gradient/curvature drift currents, an assumption that is valid only for timescales that are long compared to Alfvén wave travel times along field lines (typically a few minutes). The time dependence introduced in (7) occurs on a drift timescale, which is assumed to be much longer.
motion in the inner magnetosphere and coupling to the ionosphere to take particularly elegant forms.

2. Expressing Particle Drift and Ionosphere-Magnetosphere Coupling in Terms of Euler Potentials

[11] Substituting the definition of Euler potentials in terms of the magnetic field [Stern, 1970],

\[ B = \nabla \alpha \times \nabla \beta \]  

(8)

into equation (1), and using \( \nabla V = (\partial V/\partial \alpha)\nabla \alpha + (\partial V/\partial \beta)\nabla \beta \) and a similar expression for \( \nabla P \) leads to the particularly elegant form

\[ \frac{J_{[n]}}{B_n} - \frac{J_{[m]}}{B_m} = \frac{\partial V}{\partial \alpha} \frac{\partial P}{\partial \beta} - \frac{\partial P}{\partial \alpha} \frac{\partial V}{\partial \beta} \]  

(9)

where the subscript \( s \) refers to values evaluated just above the southern ionosphere, and \( n \) refers to the same height above the northern ionosphere. The right side of (9) can be evaluated anywhere along the field line.

[12] When the particle position is expressed in terms of Euler potentials \( (\alpha(t), \beta(t)) \), equation (4), which describes charged particle drift, takes the form

\[ \dot{\alpha} = \left( \frac{\partial \alpha}{\partial x} \right)_x + \frac{E_{ind} \times B}{B^2} \cdot \nabla \alpha - \frac{1}{q} \frac{\partial H}{\partial \beta} = - \frac{1}{q} \frac{\partial H}{\partial \beta} \]  

(10)

\[ \dot{\beta} = \left( \frac{\partial \beta}{\partial x} \right)_x + \frac{E_{ind} \times B}{B^2} \cdot \nabla \beta + \frac{1}{q} \frac{\partial H}{\partial \alpha} = \frac{1}{q} \frac{\partial H}{\partial \alpha} \]  

(11)

where the subscript \( x \) indicates that the derivative is taken at constant position \( x \). The second equality in each equation follows if we choose our gauge so that a particle drifting at velocity \( E_{ind} \times B/B^2 \) maintains constant \( \alpha \) and \( \beta \). The drift equations of motion thus correspond to those of an ideal Hamiltonian fluid.

[13] Some comment is needed concerning our treatment of induction electric fields in (10) and (11). We assume that our \( (\alpha, \beta) \) grid is fixed in the ionosphere and correspondingly neglect induction electric fields there, setting \( E = -\nabla \Phi_n \) in the northern ionosphere, for example. The motion of the ionospheric footprint of the guiding center of a magnetospheric particle moves at \( \dot{\alpha} = -(1/q)\partial H/\partial \beta, \dot{\beta} = (1/q)\partial H/\partial \alpha \). The equatorial map of a particle guiding center moves, because of the gradient of \( H \) but also because the equatorial crossing point of field line \( (\alpha, \beta) \) moves in time, at a velocity \( E_{ind} \times B/B^2 \). Thus we are not neglecting induction-electric field effects in the magnetosphere. Their effect is included in the time-dependent magnetic field mapping. We use a gauge in which the potential is constant along each magnetic field line, except for effects of field-aligned potential drops, which may be estimated from the Knight [1973] or similar algorithm.

[14] Written in terms of Euler potentials, equation (7) takes the form

\[ \left( \frac{\partial}{\partial t} + \dot{\alpha} \frac{\partial}{\partial \alpha} + \dot{\beta} \frac{\partial}{\partial \beta} \right) f = S - L \]  

(12)

In (10), (11), and (12), \( H \) and \( f \) can be regarded as functions of either \( [\lambda, \alpha, \beta, t] \) for the isotropic distribution case where \( \lambda \) is the isotropic energy invariant, or \( [\mu, J, \alpha, \beta, t] \) for the more general case where \( \mu \) and \( J \) are the first two adiabatic invariants.

[15] Our next task is to work out the generalization of equation (3), which the RCM solves for the ionospheric potential distribution, to the case where there is no symmetry between the two hemispheres and no assumption is made about constancy of the magnetic field along the conducting ionosphere regions of field lines. Using (8) to rewrite (2) in terms of Euler potentials yields the expressions

\[ J \cdot \nabla \alpha = -\frac{\partial \Phi}{\partial \alpha} \sigma_p(\nabla \alpha)^2 - \frac{\partial \Phi}{\partial \beta} (-B_{rel} + \sigma_p \nabla \nabla \beta) \]

\[ + (\nabla \nabla \alpha) \sigma_p B + (\nabla \nabla \beta) \sigma_p \]

\[ J \cdot \nabla \beta = -\frac{\partial \Phi}{\partial \alpha} (B_{rel} + \sigma_p \nabla \nabla \beta) - \frac{\partial \Phi}{\partial \beta} \sigma_p (\nabla \alpha)^2 \]

\[ + (\nabla \nabla \beta) \sigma_p B + (\nabla \nabla \alpha) \sigma_p \]

(13)

Consider a flux tube that is bounded by Euler potentials \( \alpha \) and \( \alpha + \Delta \alpha \) and also by \( \beta \) and \( \beta + \Delta \beta \), so that it carries magnetic flux \( \Delta \alpha \Delta \beta \). Use of Euler potentials facilitates expression of current conservation in a flux tube. The conduction current flowing perpendicular to \( B \) out of the northern ionosphere part of that flux tube is given by

\[ \Delta \alpha \Delta \beta \left[ \int_N \frac{J \cdot \nabla \alpha}{B} \, ds + \int_N \frac{J \cdot \nabla \beta}{B} \, ds \right] \]

That current must be balanced by the Birkeland current down into the northern ionosphere from the magnetosphere, which is given by \( J_{[n]} \Delta \alpha \Delta \beta/B_n \). Here an “\( N \)” under an integral sign indicates integration over the northern ionosphere part of a field line. The condition for current balance in the northern ionosphere is thus given by

\[ \int_S \frac{J \cdot \nabla \alpha}{B} \, ds + \int_S \frac{J \cdot \nabla \beta}{B} \, ds = J_{[n]} \]  

(14)

and the corresponding condition for the southern ionosphere is

\[ \int_S \frac{J \cdot \nabla \alpha}{B} \, ds + \int_S \frac{J \cdot \nabla \beta}{B} \, ds = -J_{[s]} \]  

(15)
The sign difference between equations (14) and (15) results from the fact that current parallel to $\mathbf{B}$ is down into the northern ionosphere but up from the ionosphere in the south.

[16] We assume that each field line is an equipotential through the conducting region of the northern and southern ionospheres, but allow a potential drop in the auroral acceleration region above each of them. Since the field-aligned currents may have different densities in the two hemispheres, we allow the potential drops to differ, letting $\Phi_n = \Phi + \Phi_{|n|}$, $\Phi_s = \Phi + \Phi_{|s|}$, and $\Delta\Phi_n = \Phi_{|n|} - \Phi_{|n|}$, where $\Phi$ is now the potential on the magnetospheric part of the field line, above both the northern and southern hemisphere auroral acceleration regions. (This is consistent with the use of the potential $\Phi$ in equations (4), (10), and (11) to move magnetospheric particles.) Adding (14) and (15) and using (13) gives the master expression that must be solved for the potential $\Phi_e$:

$$J_{|n|}/B_n - J_{|s|}/B_s = -\frac{\partial}{\partial \alpha} \left( \Sigma_{P_{|n|}} \Phi_{|n|} \right) - \frac{\partial}{\partial \beta} \left( \Sigma_{P_{|n|}} \Phi_{|n|} \right) - \frac{\partial}{\partial \alpha} \left( \Sigma_{P_{|s|}} \Phi_{|s|} \right)$$

$$- \frac{\partial}{\partial \beta} \left( \Sigma_{P_{|s|}} \Phi_{|s|} \right) + \frac{\partial \Delta\Phi_n}{\partial \alpha} - \frac{\partial \Delta\Phi_s}{\partial \alpha} + \mathcal{I}_{|n|} + W$$

(16)

where

$$\Sigma_{P_{|n|}} = \int ds \frac{\sigma_p (\nabla \alpha)^2}{B}, \quad \Sigma_{P_{|s|}} = \int ds \frac{\sigma_p (\nabla \beta)^2}{B}$$

$$\Sigma_{P_{ns}} = \int ds \frac{\sigma_p (\nabla \alpha \cdot \nabla \beta)}{B}, \quad \Sigma_{P_{HS}} = \int ds \sigma_p$$

(17)

and the integrals now include both northern and southern ionosphere ends of the field line. Equation (17) represents a generalization of equation (3) in the old formulation. The term in (16) that results from any difference between the field-aligned potential drops above the northern and southern ionospheres is given by

$$\mathcal{I}_{|n|} = -\frac{\partial}{\partial \alpha} \left( \Sigma_{P_{|n|}} \frac{\Delta\Phi_n}{\alpha} \right) - \frac{\partial}{\partial \beta} \left( \Sigma_{P_{|n|}} \frac{\Delta\Phi_n}{\beta} \right) - \frac{\partial}{\partial \alpha} \left( \Sigma_{P_{|s|}} \frac{\Delta\Phi_s}{\alpha} \right)$$

$$- \frac{\partial}{\partial \beta} \left( \Sigma_{P_{|s|}} \frac{\Delta\Phi_s}{\beta} \right) + \frac{\partial \Sigma_{P_{ns}}}{\partial \alpha} \frac{\partial \Delta\Phi_n}{\partial \alpha} - \frac{\partial \Sigma_{P_{ns}}}{\partial \beta} \frac{\partial \Delta\Phi_s}{\partial \beta}$$

(18)

where the index $S$ refers to the southern hemisphere, and $\Sigma_{P_{|n|}} = \Sigma_{P_{|n|}S} + \Sigma_{P_{|n|}N}$, etc. The wind term is given by

$$W = \sum_{i=S}^N \left( \frac{\partial J_{|n|}}{\partial \alpha} + \frac{\partial J_{|s|}}{\partial \beta} \right)$$

(19)

where the index $i$ refers to either the southern (S) or northern (N) ionosphere, and

$$J_{|n|} = \int ds \cdot \left[ \sigma_n \nabla \alpha + \frac{(\sigma_p B + \sigma_R \nabla \alpha \cdot \nabla \beta) \nabla \beta}{(\nabla \beta)^2} \right]$$

$$J_{|s|} = \int ds \cdot \left[ \sigma_s \nabla \beta + \frac{(-\sigma_p B + \sigma_R \nabla \beta \cdot \nabla \alpha) \nabla \alpha}{(\nabla \alpha)^2} \right]$$

(20)

Equations (16)–(20) represent the generalization of equation (3) to the case where neutral winds and field-aligned potential drops are included, no symmetry is assumed between hemispheres, and no assumption is made about the magnetic field being constant along the conducting region of each field line. Equations (16)–(20) are physically equivalent to equations (2.6)–(2.9) of Richmond [1995], except that we have included the effects of different field-aligned potential drops on the northern and southern hemisphere ends of the same field line; the coordinate systems used in the two cases are, of course, different (apex coordinates versus Euler potentials).

[17] For our equatorial boundary condition, we follow Richmond [1995] and set the vertical current density equal to zero at the apex of the $\alpha = 0$ field line, which just grazes the lower boundary of the conducting region. In earlier versions of the RCM, the grid has generally terminated about 10–15° from the equator, and the equatorial electrojet has been approximately represented as a conducting band. The new procedure should allow us to resolve the equatorial electrojet.

[18] The field-aligned potential drops can be estimated using the Knight [1973] relation, for example, and the values of $J_{|n|}$ and $J_{|s|}$ that were computed the previous time step. The wind velocities must come from an appropriate neutral wind model. Once the elliptic equation (16) is solved for the potential $\Phi_e$, then equations (14) and (15) can be solved (using (13)) for updated values of $J_{|n|}$ and $J_{|s|}$. If there are serious differences between the field-aligned potential drops above the northern and southern ionospheres, one could compute new values of $\Delta\Phi$ and solve (16) again for more accurate values of $\Phi_e$, $J_{|n|}$ and $J_{|s|}$.

[19] Note that this approach allows calculation of $J_{|n|}$ and $J_{|s|}$ deep in the inner magnetosphere, where the currents generated by magnetospheric pressure gradients are negligible and $J_{|n|}/B_n = J_{|s|}/B_s$. That is, the formulation allows consideration of currents that flow along field lines between the northern and southern ionospheres, driven not by magnetospheric pressure gradients but by differences between winds and conductances in the two hemispheres.

3. Numerical Calculation of Euler Potentials

[20] We begin by constructing the Euler potential grid in the northern ionosphere. Consistent with the assumption that $\mathbf{E} = -\nabla \Phi$ at ionospheric height, we assume that the radial component of the magnetic field at ionospheric altitude is independent of time, and we set it equal to the radial component of the internal magnetic field. Magnetospherically driven currents can cause variations ~1% in that component, but we neglect those effects in the present work,
where $\Lambda$ is magnetic dipole latitude, $\Lambda_{dip}$ is the dipole latitude of the dip equator, and $R_i = R_E + h_{ref}$. (Note that $B_{ir} < 0$ north of the dip equator.)

4. Set the longitudinal spacing of the $\beta$ lines by

$$d\phi \over d\beta(\phi) = \frac{2\pi}{\Lambda_{ap}(\phi)} \int_{\phi}^{\pi/2} B_p(\Lambda, \phi) \cos \Lambda d\Lambda$$

where the numerator is the total magnetic flux through the reference height divided by $R_i^2$. Equations (21) and (22) together imply that the total magnetic flux per unit $\beta$ is independent of $\beta$ (and $\phi$). Substituting (22) in (21), we find that the value of $\alpha$ at the dipole north pole, given by

$$\alpha_{\text{max}} = \frac{R_i^2}{2\pi} \int_{0}^{\pi/2} B_p(\Lambda, \phi) \cos \Lambda d\Lambda$$

which is also independent of $\phi$ (and $\beta$).

With our definition of the zero point, the Euler potential $\alpha$ is the magnetic flux per unit $\beta$ between the dip equator and the point in question. Applied to a dipole field, it follows $\alpha/\alpha_{\text{max}} = \sin^2 \Lambda$, so that we could define

$$\alpha = \alpha_{\text{max}} \sin^2 \Lambda_{eq}$$

where $\Lambda_{eq}$ is an equivalent dipole latitude.

In the RCM implementation, we calculate the integrals in (21) and (22) on a very dense grid, much finer than the one we use for the full RCM, and then calculate the locations of the actual RCM grid points using (22) and (21) by interpolation. A sample RCM grid is shown in Figure 1, computed assuming an IGRF-2000 magnetic field. Figure 2 shows several aspects of our choice for the latitudinal grid spacing, specifically showing $\alpha$, latitude, and apex altitude versus latitude grid index $i$. The latter two quantities actually depend on $\beta$: the single curves in Figures 2b and 2c were computed assuming a dipole field and are therefore only approximate.

Note the following:

1. In Figure 1, the outer boundary of the grid is not a circle, because the distance from the magnetic pole to the dip equator varies with longitude.

2. The constant-$\alpha$ grid curves in Figure 1 are smooth near the pole, as a result of our use of (22) to define the spacing between adjacent constant-$\beta$ curves.

3. We chose to make the latitudinal grid spacing relatively tight both at low latitudes and in the auroral zone. The low-latitude grid spacing was chosen to so that the apex heights at adjacent grid points are close enough to resolve the equatorial E and $F$ regions (right side of Figure 2). The spacing was made tight in the auroral zone (approximately 0.4° to 1°) to resolve that crucial region. Of course, the details of the distribution are easily adjusted, and the number of grid points can be increased if necessary.
Once the northern ionosphere grid is determined, the southern ionosphere grid is calculated by numerically tracing field lines from the northern hemisphere at the reference altitude to the southern ionosphere at the same altitude. In the northern ionosphere, the $\beta = \text{constant}$ lines were arbitrarily assumed to be curves of constant dipole longitude. The same is generally not true of the corresponding grid in the southern ionosphere.

Though it is reasonable to consider just the Earth’s internal magnetic field to trace field lines through either the northern or southern ionosphere, that approximation is inadequate for the purpose of tracing field lines through the magnetosphere between northern and southern ionospheres. A magnetospheric magnetic field model is required for that purpose. In recent event simulations with the RCM [e.g., Garner et al., 2004; Sazykin et al., 2005], we have normally let the magnetic field vary in time, using the Hilmer and Voigt [1995] or Tsyganenko [2000] and Tsyganenko et al. [2003] models to precompute magnetic field-mapping information at a series of mark times through the event (e.g., every 10 min); to let the magnetic field vary continuously in time, we reinterpolate every time step, between the nearest two mark times. For some recent runs in which the RCM has been coupled to codes that compute magnetic fields that are in pressure balance [e.g., Lemon et al., 2004; De Zeeuw et al., 2004], magnetic fields have been recomputed self-consistently at frequent intervals through events. In the new RCM, the values of the conductance and wind integrals (17 and 20) are recomputed by numerical integration every magnetic field mark time. Depending on the situation, the values that are used in (16) are either held constant between mark times, or linearly interpolated in time.

It should be noted that our procedure for constructing a southern ionosphere grid based on Euler potentials does not include the open field line region of the southern ionosphere. This is illustrated in Figure 3, which shows the result of using a T96_01 magnetic field model [Tsyganenko and Stern, 1996] to map the Northern hemisphere grid of Figure 1 to the Southern hemisphere. The central region where no grid lines are shown corresponds to the region of open field lines in the T96 model. It is also clear from Figure 3 that the southern hemisphere Euler potential grid becomes irregular near the open-closed boundary, because of the near-singular mapping in that region. This problem occurs poleward of the modeling region of the RCM and other inner-magnetosphere models, but it does represent a problem for constructing a global ionospheric grid based on Euler potentials. Our procedure does not produce a useful grid in and near the southern polar cap.

One might argue that global-MHD codes treat the coupling of the two hemispheres in a much simpler and more natural way. They deal with coupling of the magnetosphere to the southern ionosphere independently of the coupling to the northern ionosphere, relying on the equi-

**Figure 2.** Details on latitudinal grid spacing. (a) $\alpha/\alpha_{\text{max}}$ as a function of grid index $i$. (b) Approximate latitudinal spacing and (c) approximate magnetic apex heights of the first 25 grid points.

**Figure 3.** Southern ionosphere grid computed for a Tsyganenko and Stern [1996] magnetic field model. Every third latitudinal grid point is shown. The orientation of the Earth is the same as in Figure 1, and the Sun is to the right.
tions of ideal MHD to enforce equipotentiality of the magnetic field lines. However, that approach is subject to numerical error and sometimes allows substantially different potential drops between the equatorial magnetosphere and the northern and southern ionospheres. Our procedure allows specification of these potential drops directly from an explicit physical algorithm (e.g., Knight relation), rather than allowing them to be governed by numerical error.

4. Summary and Discussion

[36] This paper presents a new formulation of the basic equations of magnetosphere-ionosphere coupling, a formulation that allows inner-magnetosphere convection models to be generalized beyond the earlier simplifying assumption of an aligned dipole, zero-tilt representation of Earth’s internal magnetic field. The new formulation facilitates proper consideration of seasonal and IMF-\(B_y\) effects in the magnetosphere as well as seasonal and longitude effects in the ionosphere. The new formulation utilizes a computational mesh based on Euler potentials, allowing the adiabatic drift and Birkeland current equations to be written in an elegant form. The expressions for conservation of ionospheric currents also are most naturally derived and expressed in terms of Euler potentials. Our procedure has the disadvantage that it does not produce a grid in the polar cap region of the southern ionosphere, but that does not limit its usefulness for models of inner-magnetospheric convection and the ring current.

[37] Acknowledgments. This research has been supported by the Upper Atmosphere Sciences Section of the National Science Foundation, under grant ATM-0334400 to Rice and grant ATM-0334732 to Prairie View A&M. The work has also received partial support from the Center for Integrated Space Weather Modeling (CISM), which is funded by the STC Program of the National Science Foundation under agreement ATM-0334400 to Rice and grant ATM-0334372 to Prairie View, TX 77446, USA. The work has also received partial support from the Center for Integrated Space Weather Modeling (CISM), which is funded by the STC Program of the National Science Foundation under agreement ATM-0334400 to Rice and grant ATM-0334372 to Prairie View, TX 77446, USA.

[38] Zuyin Pu thanks George Siscoe and Paul Song for their assistance in evaluating this paper.

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